AD A124897

(12)

Report Number 3

A PRONY METHOD FOR NOISY DATA: CHOOSING THE SIGNAL COMPONENTS AND SELECTING THE ORDER IN EXPONENTIAL SIGNAL MODELS

Ramdas Kumaresan
Donald W. Tufts
Louis L. Scharf

Department of Electrical Engineering
University of Rhode Island
Kingston, Rhode Island 02881

December 1982

Prepared for

OFFICE OF NAVAL RESEARCH (Code 411SP)
Statistics and Probability Branch
Arlington, Virginia 22217
under Contract N00014-81-K-0144
D. W. Tufts, Principal Investigator
and Contract N00014-82-K-0300
L. L. Scharf, Principal Investigator

DTIC
ELECTE
FEB 23 1983
S D
A

Approved for public release; distribution unlimited

DTIC FILE COPY

83 02 023 140

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: 3	2. GOVT ACCESSION NO.: AD-A124 897
	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): A Prony Method for Noisy Data: Choosing the Signal Components and Selecting the Order in Exponential Signal Models	5. TYPE OF REPORT & PERIOD COVERED: April 1982 - Sept. 1982
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Ramdas Kumaresan, Donald W. Tufts, Louis L. Scharf	8. CONTRACT OR GRANT NUMBER(s): N00014-81-K-0144, N00014-82-K-0300
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Electrical Engineering, University of Rhode Island, Kingston, Rhode Island 02881	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research (Code 411 SP), Department of the Navy, Arlington, Virginia 22217	12. REPORT DATE: December 1982
	13. NUMBER OF PAGES: 17
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Prony's Method, Exponential Model Fitting, Order Selection, Subset Selection, High-Resolution Signal Processing, Linear Prediction, Estimation of Signal Parameters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Prony's method is a simple procedure for determining the value of parameters of a linear combination of exponential functions. Until recently, even the modern variants of this method have performed poorly in the presence of noise. We propose a simple procedure for estimation of the signal parameters in the presence of noise. This procedure is very close in form and assumptions to Prony's method. However, in preliminary tests, the performance of the method is close to that of the best available, more

(Abstract con't.)

complicated, approaches which are based on maximum likelihood or on the use of eigenvector or singular vector decompositions.

A Prony Method for Noisy Data: Choosing the Signal Components and Selecting the Order in Exponential Signal Models

Ramdas Kumaresan

Donald W. Tufts

Louis L. Scharf

Department of Electrical Engineering
Kelley Hall
University of Rhode Island
Kingston, R.I. 02881

ABSTRACT

Prony's method is a simple procedure for determining the values of parameters of a linear combination of exponential functions. Until recently, even the modern variants of this method have performed poorly in the presence of noise. We propose a simple procedure for estimation of the signal parameters in the presence of noise. This procedure is very close in form and assumptions to Prony's method. However, in preliminary tests, the performance of the method is close to that of the best available, more complicated, approaches which are based on maximum likelihood or on the use of eigenvector or singular vector decompositions.

DTIC COPY INSPECTED 2

Accession For
NTIS GRA&I
DTIC TAB
Unannounced
Justification
Distribution/
Availability Codes
Avail and/or
Dist Special
A

I. Introduction

Nearly two hundred years ago Prony developed a simple procedure for determining the values of parameters of a linear combination of exponential functions from uniformly spaced samples [1]. Today "Prony's method" is usually taken to mean the least squares extension of the method as presented, for example, by Hildebrand [2].

A short record of a data sequence y(n), n = 1,2,...N, is assumed to be composed of uniformly spaced samples of a sum of exponential signals, x(n), and measurement noise w(n). That is,

$$y(n) = x(n) + w(n) \quad \text{for } n = 1,2,\ldots,N \tag{1}$$

where

$$x(n) = \sum_{k=1}^{M} a(k)(C(k))^{n} \tag{2}$$

$$N > 2M \tag{3}$$

$$C(k) = \exp(s(k)) \tag{4}$$

The values of the signal parameters a(k) and s(k) for K = 1,2,...,M are unknown complex numbers. Often the value of M is unknown also. However, let us initially assume that the value of M is known.

Following the derivation of Hildebrand [2] we note that the signal x(n) satisfies a linear, homogeneous difference equation with constant coefficients

$$\sum_{k=0}^{M} b(k)\, x(n-k) = 0 \quad \text{for} \quad M < n \leq N \tag{5}$$

where

$$b(0) = 1 \tag{6}$$

The roots of the prediction-error-filter polynomial B(z) provide the values of

the exponent parameters C(k), and hence s(k):

$$B(z) = \sum_{k=0}^{M} b(k)z^{-k}$$

$$= \prod_{k=1}^{M} \left[1-C(k)z^{-1}\right] \tag{7}$$

Hildebrand explicitly considers noisy data and specifies Prony's method by the following three steps:

(1) Using the method of least squares, minimize the approximation error

$$A = \sum_{n=M+1}^{N} \left| \sum_{k=1}^{M} \hat{b}(k)\, y(n-k) + y(n) \right|^2 \tag{8}$$

by best choice of the coefficients $\hat{b}(k)$ [3]. For N > 2M and for noisy data the solution will be unique with high probability. However, if the resulting set of normal equations is singular, then the pseudo inverse of the coefficient matrix can be used to choose the minimum norm solution.

(2) After the M values of $\hat{b}(k)$ are determined, the roots of the prediction-error-filter polynomial, $\hat{B}(Z)$, are found, using $\hat{b}(0) = 1$.

$$\hat{B}(z) = \sum_{k=0}^{M} \hat{b}(k)z^{-k} = \prod_{k=1}^{M} (1 - \hat{C}(k)z^{-1}) \tag{9}$$

The corresponding exponent values, $\hat{s}(k)$, can then be found from formula (4).

(3) Having determined the values $\hat{C}(k)$ for k = 1,2,...,M, the error in approximating the observed data by a linear combination of exponential signal components then becomes linear in the M values of a(k):

$$e(n) = y(n) - \sum_{k=1}^{M} a(k) \left[\hat{C}(k)\right]^n \quad \text{for} \quad n = 1,2,\ldots N \tag{10}$$

The M estimates, $\hat{a}(k)$, can be determined by minimizing the summed, magnitude-squared error:

$$E = \sum_{n=1}^{N} \left| y(n) - \sum_{k=1}^{M} a(k) \left[\hat{C}(k)\right]^{n} \right|^{2} \tag{11}$$

It is well known that the errors in signal parameters which are estimated by Prony's method can be discouragingly large [2,4] . For insight into this phenomenon we recommend calculation and study of the Cramer-Rao (CR) bounds for the variance of the error in the estimated parameters and comparison of the threshold of estimation of the Prony method with that of the maximum likelihood method. [7,8,9,10]. By the threshold of estimation, we mean the value of signal-to-noise ratio (SNR) at which the variance of an estimation error begins to depart very rapidly from the corresponding CR-bound value.

As another example of the application of Prony's method consider the problem of estimating the parameters of a zero-mean, autoregressive, moving average(ARMA) stationary random sequence from estimates of its covariance values. Various investigators have recognized that, after a finite number of lags, the true underlying covariance values satisfy a linear, homogeneous, difference equation with constant coefficients [11,12,13]. That is, after a finite number of lags, the estimated covariance sequence can be represented as a linear combination of exponentials (i.e. the true, underlying covariance sequence which satisfies the homogeneous difference equation) plus measurement noise. This measurement noise may be only the error sequence in estimating the covariance values from a finite observation of the ARMA sequence. Part of it might also be due to additive noise in the observation of the ARMA sequence. Some of these ideas have been restated by Cadzow [14, 15].

II. Prony Methods for Noisy Data

In previous and related work, we have shown how one can extend the threshold of estimation of Prony's method to much lower values of SNR and how one can

improve parameter estimation at values of SNR above this threshold [8,9,10,11,16-22]. The major source for these improvements is the use of information about the rank, M, of a matrix of signal covariance values or a matrix of samples of the signal. If there is no prior information about this rank, it is estimated from the data using singular value decomposition (SVD). The most important computational step is a preprocessing step, before application of Prony's method. A prediction order L which is larger than the value of M is chosen. The measured data matrix or the matrix of estimated covariance values is replaced by a matrix of the prescribed rank, M, which is the best least squares approximation to the given matrix. Other investigators have presented closely related approaches [23-29].

In this work we advocate a simpler procedure which appears to provide the same desirable attributes to nearly the same extent. This procedure consists of the following two steps:

(1) Use Prony's method on the given data, but with a prediction order L which is larger than the maximum number of exponentials which are expected in the signal. The result is a set of L exponentials which are candidates for signal components.

(2) Out of the L exponential functions which are provided by the high-order Prony calculation, determine the best subset of size M. A best subset of M exponentials is one for which a linear combination of the M exponentials best approximates the observed data using a least squares criterion. One can check all of the $\binom{L}{M}$ possible subsets of size M of the L exponential functions to find the best combination.

A simpler approach to step 2 is to use the procedure of Hocking and Leslie [30] as we have previously suggested [31]. In the procedure of

Hocking and Leslie a best subset can usually be found without searching over all possible subsets. Hocking and Leslie accomplish this by first solving a related, but different, problem. They search for the basis functions (exponentials, in our application) which contribute most to the summed magnitude-squared errors by their deletion. This provides an initial importance ordering of the exponentials. Hocking and Leslie prove that the fitting errors associated with these single-deletion sets provide convenient threshold values of the error for recognizing the global optimality of a particular combination of M exponentials being tested.

If M is not known apriori, an estimate of M, ($\hat{M}$) can be found as follows. Choose $\hat{M} = 1$, and find the best subset of size unity that best fits the data. Call the corresponding minimum error, E_1. Then, choose $\hat{M} = 2$ and find the best subset of size two and the corresponding minimum error E_2. Repeat the procedure until the rate of decrease of the error with increasing values of $\hat{M}$ is small, consistent with the modeling of broadband noise. The integer i at which E_i shows the significant drop in rate of decrease is taken as $\hat{M}$. We now give a simulation example.

III. Simulation Results

If the data is known to be composed of undamped sinusoids, as we assume in this example, forward and backward prediction equations can be used simultaneously to obtain extra prediction equations for Hildebrand's least squares form of the Prony method [32,33,34].

A sequence y(n) consisting of two complex sinusoids and white, complex Gaussian noise w(n) was generated using the formula below:

$$y(n) = a_1 e^{j(\omega_1 n + \phi_1)} + a_2 e^{j(\omega_2 n + \phi_2)} + w(n), \quad n = 0,1,2\ldots 24$$

Here $a_1 = a_2 = 1$, $\omega_1 = 2\pi(0.52)$, $\omega_2 = 2\pi(0.5)$ and $j = \sqrt{-1}$. The variance of

the real or imaginary part of w(n) is σ^2. SNR is defined as 10 $\log_{10}$ $(a_1^2/2\sigma^2)$. The coefficients of the polynomial B(z) were found by solving the forward-backward linear prediction equations as in ref. [35] in the least-squares sense. L was chosen to be 12 (N/2). The 12 zeros of G(z) were found and the best subset of 2 out of the 12 which minimized E in formula 11 was computed. The frequency estimates of the two sinewaves, $\hat{f}_1$ and $\hat{f}_2$ are the angles of the two chosen exponents (divided by 2π). This simulation was repeated 500 times and the root mean square (RMS) value of the frequency estimation error was computed at SNR values in the range of 30dB to 7dB. They are given in Table 1 along with the appropriate CR bounds and SVD-method values which were taken from ref. (35). Comparing these figures with those in [35], we note that the SVD-based methods are slightly better in performance. This difference is due to the signal enhancement achieved by SVD.

Figure 1 shows the minimum subset error $E_{\hat{M}}$ for different choices of $\hat{M}$ at different SNR values. The value E_0 at $\hat{M} = 0$ is the data "energy" $\sum_{n=1}^{N} |y(n)|^2$. Note the clear drop in E at $\hat{M} = 2$.

IV. Discussion and Conclusions

Ideally, to fit exponentials to a data sequence y(n), one has to minimize the error $\sum_{n=1}^{N} \left| y(n) - \sum_{k=1}^{M} \hat{a}_k e^{\hat{s}_k n} \right|^2$ with respect to $\hat{a}_k$'s and $\hat{s}_k$'s simultaneously. This is a difficult problem even if the value of M is known. Instead, we find the exponents $\hat{s}(k)$ separately as is often done. However, we have made use of the fact that, if the data is composed of exponentials and noise, overestimating the degree L (> M) of the polynomial B(z) improves the accuracy of the M signal-zero locations. Subsequently, we select the M out of the L exponentials that best explain the data. The new procedure extends the threshold of the forward-backward covariance method [32,33,34] and is only slightly inferior to SVD based methods (8,35).

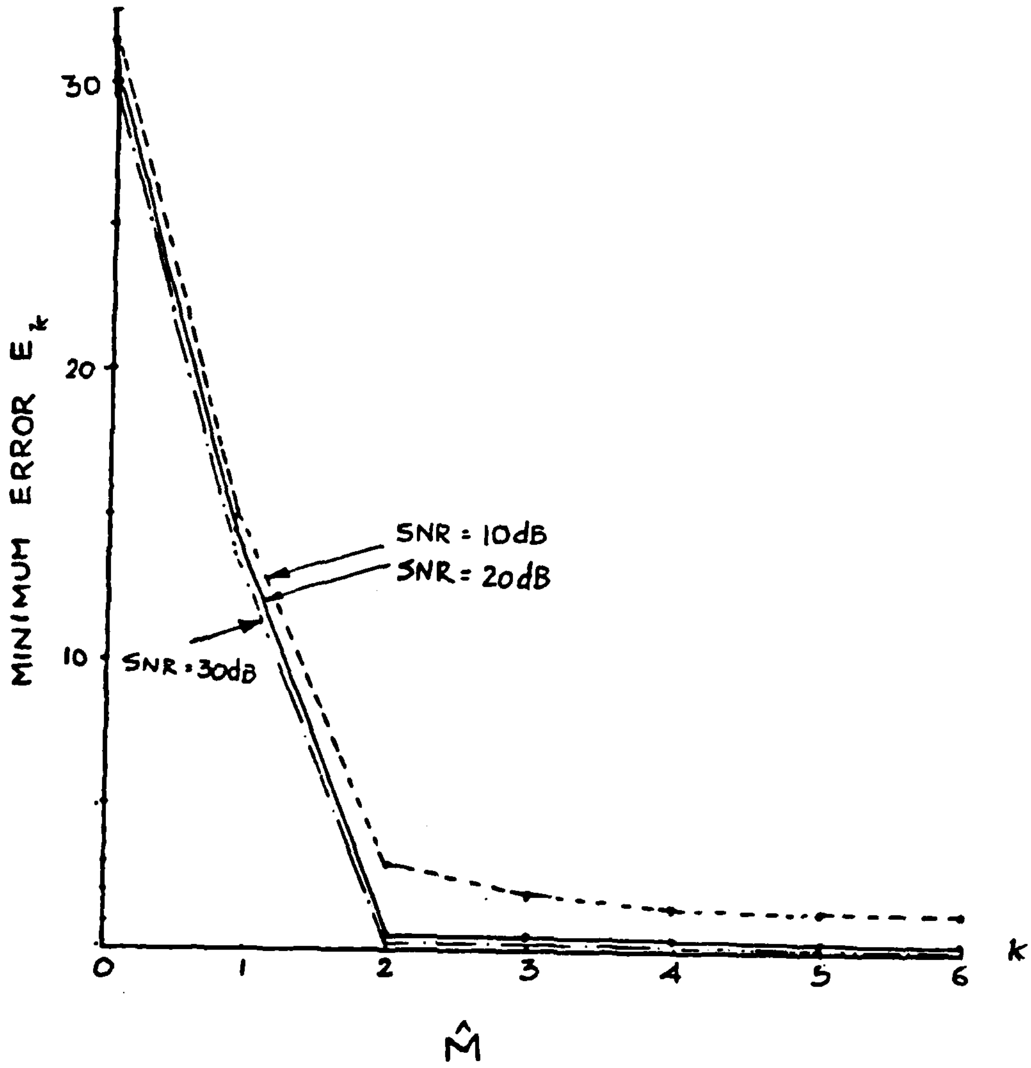

FIGURE 1 Minimum Subset Error Versus Hypothesized Subset Size (the Estimated Number of Exponential Signal Components)

SNR dB	$\sqrt{\text{Mean Square Error}}$	CRB
30	0.427×10^{-3} (SVD) 0.403×10^{-3} ($L = 12$)	0.311×10^{-3}
20	0.130×10^{-2}	0.984×10^{-3}
15	0.238×10^{-2}	0.175×10^{-2}
12	0.373×10^{-2} (SVD) 0.313×10^{-2} ($L = 12$)	0.276×10^{-2}
10	0.417×10^{-2}	0.311×10^{-2}
7	0.601×10^{-2}	0.490×10^{-2}

TABLE 1: Mean square error of the frequency (f_1) estimation error vs. SNR. CRB stands for the Cramer-Rao bound which is the lower bound on the standard deviation of the frequency estimation error for an unbiased estimator. The bias in the frequency estimates was insignificant except at SNR = 7dB. Below 7dB the mean square error is large due to the presence of outliers. For the proposed subset selection method the prediction order is twelve ($L = 12$). For comparison, two values of the error for the SVD method are provided [35].

References

1. R. de Prony, "Essai Experimentale et Analytique," J. Ecole Polytechnique (Paris), pp. 24-76, 1795.

2. F.B. Hildebrand, Introduction to Numerical Analysis, pp. 378-382, McGraw-Hill Book Company, New York, 1956.

3. F.B. Hildebrand, Op. Cit., formula 9.4.6, p. 379.

4. C. Lanczos, Applied Analysis, pp. 272-280, Prentice-Hall, Inc., 1956.

5. L.E. Brennan, "Angular Accuracy of a Phased Array Radar," IRE Trans. Antennas Propagat., Vol. AP-9, pp. 268-275, May 1961.

6. D.C. Rife and R.R. Boorstyn, "Multiple Tone Parameter Estimation from Discrete Time Observations," Bell Syst. Tech. J., pp. 1389-1410, Nov. 1976.

7. D.W. Tufts and R. Kumaresan, "Improved Spectral Resolution II," Proc. ICASSP 80, pp. 592-597, Apr. 1980.

8. D.W. Tufts and R. Kumaresan, "Estimation of Frequencies of Multiple Sinusoids: Making Linear Prediction Perform Like Maximum Likelihood," Proc. IEEE, Vol. 70, pp. 975-989, Sept. 1982.

9. Ramdas Kumaresan and D.W. Tufts, "Accurate Parameter Estimation of Noisy Speech-Like Signals," Proceedings of the 1982 IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP 82), pp. 1357-1361.

10. Ramdas Kumaresan and D.W. Tufts, "Estimating the Parameters of Exponentially Damped Sinusoids and Pole-Zero Modeling in Noise," to appear in IEEE Transactions on Acoustics, Speech, and Signal Processing.

11. W. Gersh, "Estimation of the Autoregressive Parameters of a Mixed Autoregressive Moving Average Time Series," IEEE Trans. Auto. Contr., AC-15, p. 583 (October 1970).

12. R.K. Mehra, "On-Line Identification of Linear Dynamic Systems with Applications to Kalman Filtering," IEEE Trans. Auto. Contr., AC-16, p. 12 (Feb. 1971).

13. A.A. Beex and L.L. Scharf, "Covariance Sequence Approximation for Parametric Spectrum Modelling," IEEE Trans. Acoust., Speech, Sign. Proc., ASSP-29, p. 1042 (October 1981).

14. J.A. Cadzow, "Spectral Estimation: An Overdetermined Rational Model Equation Approach," Proc. IEEE, Vol. 70, pp. 907-939, Sept. 1982.

15. J.A. Cadzow, "High Performance Spectral Estimation - A New ARMA Method," IEEE Trans. Acoustics, Speech, and Signal Processing, Vol. ASSP-28, pp. 524-529, October 1980.

16. Ramdas Kumaresan and D.W. Tufts, "Improved Spectral Resolution III: Efficient Realization," Proceedings Letters, Proc. IEEE, Vol. 68, pp. 1334-1335, October 1980.

17. Ramdas Kumaresan and D.W. Tufts, "Data-Adaptive Principal Component Signal Processing," Proceedings of the 19th IEEE Conference on Decision and Control, Albuquerque, N.M., December 1980.

18. Ramdas Kumaresan and D.W. Tufts, "Singular Value Decomposition and Spectral Analysis," Proceedings of the First IEEE ASSP Workshop on Spectral Estimation, Hamilton, Ontario, pp. 6.4.1 to 6.4.12, August 1981.

19. Ramdas Kumaresan and D.W. Tufts, "A Two-Dimensional Technique for Frequency-Wavenumber Estimation," Proceedings Letters, Proc. IEEE, Vol. 69, pp. 1515-1517, Nov. 1981.

20. Ramdas Kumaresan and D.W. Tufts, "Singular Value Decomposition and Spectral Analysis," Proceedings of the 20th IEEE Conference on Decision and Control, San Diego, pp. 1-11, Dec. 1981.

21. D.W. Tufts, Ramdas Kumaresan and Ivars Kirsteins, "Data-Adaptive Signal Estimation," Proceedings of the Sixteenth Annual Princeton Conference on Information Sciences and Systems in March 1982 and Proceedings Letters, Proc. IEEE, Vol. 70, pp. 684-685, June, 1982.

22. R. Kumaresan, "Accurate Frequency Estimation Using an All-Pole Filter with Mostly Zero Coefficients," Proceedings of the IEEE, Vol. 70, No. 8, August 1982.

23. T.L. Henderson, "Geometric Methods for Determining System Poles from Transient Response," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, pp. 982-988, October, 1981.

24. S.S. Reddi, "Multiple Source Location - A Digital Approach," IEEE Trans. on Aero. and Elec. Syst., Vol. AES-15, No. 1, pp. 95-105.

25. A. Cantoni and L.C. Godara, "Resolving the Directions of Sources in a Correlated Field Incident on an Array," J. of Acous. Soc. Am., 67(4), April 1980, pp. 1247-1255.

26. W.S. Liggett, "Passive Sonar: Fitting Models to Multiple Time Series," in Signal Processing, Ed. J.W.R. Griffiths et al., Academic Press, New York, 1973.

27. N.L. Owsley, "A Recent Trend in Adaptive Spatial Processing for Sensor Arrays: Constrained Adaptation," in Signal Processing, Ed. J.W.R. Griffiths et al., Academic Press, New York, 1973.

28. G. Bienvenu and L. Kopp, "Adaptivity to Background Noise Spatial Coherence for High Resolution Passive Methods," Proc. of ICASSP 1980, April 1980, pp. 307-310.

29. R. Schmidt, "Multiple Emitter Location and Signal Parameter Estimation," Proc. RADC Spectral Estimation Workshop, pp. 243-258, Rome, NY, 1979.

30. R.R. Hocking and L.L. Leslie, "Selection of the Best Subset in Regression Analysis," Technometrics, Vol. 9, pp. 537-540, 1967.

31. D.W. Tufts and R. Kumaresan, "Improved Spectral Resolution," Proceedings Letters, Proc. IEEE, Vol. 68, pp. 419-420, March, 1980.

32. A.H. Nuttall, "Spectral Analysis of a Univariate Process with Bad Data Points via Maximum Entropy and Linear Predictive Techniques," in NUSC Scientific and Engineering Studies, Spectral Estimation, NUSC, New London, CT, March, 1976.

33. T.J. Ulrych and R.W. Clayton, "Time Series Modelling and Maximum Entropy," Physics of the Earth and Planetary Interiors, Vol. 12, pp. 188-200, August, 1976.

34. S.W. Lang and J.H. McClellan, "Frequency Estimation with Maximum Entropy Spectral Estimators," IEEE Trans, on ASSP, Vol. 28, No. 6, Dec. 1980, pp. 716-724.

35. D.W. Tufts and Ramdas Kumaresan, "Singular Value Decomposition and Improved Frequency Estimation Using Linear Prediction," IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-30, pp. 671-675, August, 1982.

OFFICE OF NAVAL RESEARCH
STATISTICS AND PROBABILITY PROGRAM

BASIC DISTRIBUTION LIST
FOR
UNCLASSIFIED TECHNICAL REPORTS

FEBRUARY 1982

	Copies
Statistics and Probability Program (Code 411(SP)) Office of Naval Research Arlington, VA 22217	3
Defense Technical Information Center Cameron Station Alexandria, VA 22314	12
Commanding Officer Office of Naval Research Eastern/Central Regional Office Attn: Director for Science Barnes Building 495 Summer Street Boston, MA 02210	1
Commanding Officer Office of Naval Research Western Regional Office Attn: Dr. Richard Lau 1030 East Green Street Pasadena, CA 91101	1
U. S. ONR Liaison Office - Far East Attn: Scientific Director APO San Francisco 96503	1
Applied Mathematics Laboratory David Taylor Naval Ship Research and Development Center Attn: Mr. G. H. Gleissner Bethesda, Maryland 20084	1
Commandant of the Marine Coprs (Code AX) Attn: Dr. A. L. Slafkosky Scientific Advisor Washington, DC 20380	1
Navy Library National Space Technology Laboratory Attn: Navy Librarian Bay St. Louis, MS 39522	1
U. S. Army Research Office P.O. Box 12211 Attn: Dr. J. Chandra Research Triangle Park, NC 27706	1
Director National Security Agency Attn: R51, Dr. Maar Fort Meade, MD 20755	1
ATAA-SL, Library U.S. Army TRADOC Systems Analysis Activity Department of the Army White Sands Missile Range, NM 88002	1
ARI Field Unit-USAREUR Attn: Library c/o ODCSPER HQ USAEREUR & 7th Army APO New York 09403	1
Library, Code 1424 Naval Postgraduate School Monterey, CA 93940	1
Technical Information Division Naval Research Laboratory Washington, DC 20375	1
OASD (I&L), Pentagon Attn: Mr. Charles S. Smith Washington, DC 20301	1

Copies	
Director AMSAA Attn: DRXSY-MP, H. Cohen Aberdeen Proving Ground, MD 21005	1
Dr. Gerhard Heiche Naval Air Systems Command (NAIR 03) Jefferson Plaza No. 1 Arlington, VA 20360	1
Dr. Barbara Bailar Associate Director, Statistical Standards Bureau of Census Washington, DC 20233	1
Leon Slavin Naval Sea Systems Command (NSEA 05H) Crystal Mall #4, Rm. 129 Washington, DC 20036	1
B. E. Clark RR #2, Box 647-B Graham, NC 27253	1
Naval Underwater Systems Center Attn: Dr. Derrill J. Bordelon Code 601 Newport, Rhode Island 02840	1
Naval Coastal Systems Center Code 741 Attn: Mr. C. M. Bennett Panama City, FL 32401	1
Naval Electronic Systems Command (NELEX 612) Attn: John Schuster National Center No. 1 Arlington, VA 20360	1
Defense Logistics Studies Information Exchange Army Logistics Management Center Attn: Mr. J. Dowling Fort Lee, VA 23801	1

Copies	
Reliability Analysis Center (RAC) RADC/RBRAC Attn: I. L. Krulac Data Coordinator/ Government Programs Griffiss AFB, New York 13441	1
Technical Library Naval Ordnance Station Indian Head, MD 20640	1
Library Naval Ocean Systems Center San Diego, CA 92152	1
Technical Library Bureau of Naval Personnel Department of the Navy Washington, DC 20370	1
Mr. Dan Leonard Code 8105 Naval Ocean Systems Center San Diego, CA 92152	1
Dr. Alan F. Petty Code 7930 Naval Research Laboratory Washington, DC 20375	1
Dr. M. J. Fischer Defense Communications Agency Defense Communications Engineering Center 1860 Wiehle Avenue Reston, VA 22090	1
Mr. Jim Gates Code 9211 Fleet Material Support Office U. S. Navy Supply Center Mechanicsburg, PA 17055	1
Mr. Ted Tupper Code M-311C Military Sealift Command Department of the Navy Washington, DC 20390	1

Copies

Mr. F. R. Del Priori
Code 224
Operational Test and Evaluation
Force (OPTEVFOR)
Norfolk, VA 23511 1